On the Secular Acceleration of the Earth's Orbital Motion.
By P. H. Cowell, M.A.

In a paper in the Supplementary Number of the *Monthly Notices* I showed that the ancient solar eclipses are satisfied by adopting the following secular terms:

1. In the distance of the Moon from the node $+4''\cdot4$
2. In the distance of the Moon from the Sun $+6''\cdot8$

and this month, in a separate paper, I show that these conclusions are supported by the Ptolemaic eclipses of the Moon recorded in the Almagest.

I then reasoned in the following way: The Sun must be assumed to have no sidereal secular acceleration. Therefore the sidereal secular acceleration of the Moon is $+6''\cdot8$ and of the node $+2''\cdot4$. On sending this result to Professor Newcomb, I received a reply alluding sarcastically to the "beautiful consistency . . . . for a result contravening gravitational theory." Professor Newcomb is quite right, and I can only express my obligation to him for putting his arguments as forcibly as he did.

A day or two later the following modification of my argument suggested itself to me. Let us accept the value $+6''\cdot5$ as the sidereal secular acceleration of the node. It follows then from the ancient eclipses that $+10''\cdot9$ is the sidereal acceleration of the Moon and $+4''\cdot1$ that of the Sun.

This alteration of my conceptions only affects one part of the numerical calculations of my late paper on solar eclipses, and that part only to an unimportant degree. The formulæ I gave for the longitude of the node and the longitude of the Sun were intended to represent these longitudes measured from the equinox. They really represent the longitudes measured from an arbitrary departure point $+4''\cdot1T^2$ in advance of the equinox. With this change of definition no figure in the calculations requires alteration until we come to the corrections for parallax. Now the effect of substituting the true equinox for the departure point, which is about one-hundredth part of a radian away from it, can at the outside only affect the corrections for parallax by 1 per cent. or $20''$. Corrections of this magnitude are unimportant.

One other change is required. Instead of putting $\Delta V = T^2 s_L$, I should have put $\Delta(V - V') = T^2 . s_D$, where s_D is the correction to the assumed secular acceleration of the mean elongation.

A secular acceleration of the Earth's orbital motion does not "contravene gravitational theory," for it may be ascribed to the resistance of the ether. The sum of the kinetic and potential energies of the Earth in its orbit is equal to the potential energy it would have at a distance from the Sun equal to twice

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its mean distance. A secular acceleration of $1''$ per century, or an increase per century of $2''$ in its centennial motion, or one part in 64,800,000, corresponds with a decrease of the mean distance of one part in 97,200,000. The loss of energy is therefore equal to the attraction of the Sun at double mean distance multiplied by twice the above fraction of the mean distance, which is equal to the attraction of the Sun at mean distance multiplied by the mean distance and divided by 194,400,000. If this loss of energy is sustained by the Earth in passing through 100 circumferences of its orbit, we conclude that the retarding force corresponding to a secular acceleration of a second is equal to the attraction of the Sun divided by $194,400,000 \times 2\pi \times 100$. For the secular acceleration found by me the divisor is 3×10^{10} nearly.

In order to obtain the theoretical secular acceleration of the node, Professor Newcomb's advice to me was: "Take Brown's mean motion of the node and differentiate it." The calculation is an easy one and goes as follows:

According to Professor Brown (*Monthly Notices*, vol. lxiv. p. 532) the centennial motion of the node contains the following terms, proportional to the square of the solar eccentricity:

$$-2546-57+8=2595''$$

From Professor Newcomb's tables of the Sun the solar eccentricity contains a factor

$$1-T\times 0{\cdot}002507-T^2 0{\cdot}00000752$$

T being measured in centuries from 1850, and its square therefore contains the factor

$$1-T\times 0{\cdot}005014-T\ 0{\cdot}00000875$$

and the mean motion of the node the variable part

$$+T\times 13''{\cdot}01+T^2\times 0''{\cdot}023$$

the longitude of the node (measured from a fixed point, and not a moving equinox) contains terms

$$+T^2\times 6''{\cdot}50+T^3\times 0''{\cdot}008$$

where the epoch is 1850, but may be taken as 1800 without sensible error.

By Mr. Crommelin's suggestion (I owe much to his many suggestions, that are based on a very wide acquaintance with astronomical literature) I have examined the transits of *Mercury* with a view to ascertaining whether the secular acceleration of the Earth is supported by them. I find on solving the equations of condition

secular acceleration of *Mercury*	...	...	$0''\cdot0$
secular acceleration of Earth	...	...	$3''\cdot0$

Details will shortly be published. It will be noted that the two entirely distinct methods of deducing a secular acceleration for the Earth are in close accordance. The zero result for *Mercury* is an argument against the cause being the resistance of the ether.

On the Ptolemaic Eclipses of the Moon recorded in the Almagest. By P. H. Cowell, M.A.

The nature of the changes in the secular terms of the lunar and solar tables now in use, that I am advocating, may be described as follows :—

1. A correction of $-0''\cdot6$ to the secular term of the longitude of the node to bring it into accordance with Professor Brown's calculations.

2. A correction of $-1''\cdot6$ to the secular term in the mean elongation.

3. A correction of $+3''\cdot0$ to the secular term in the argument of latitude.

In order to see what light the ancient lunar eclipses throw upon these corrections it will be remarked :

1. The position of the equinox does not enter into a lunar eclipse. We can only hope, therefore, to obtain from lunar eclipses the relative positions of the Sun, Moon, and node, and we cannot hope to get the position of the equinox in addition.

2. The present *Nautical Almanac* is based upon Professor Newcomb's researches. Mr. Nevill has already shown (*Monthly Notices*, vol. xxxix.) that the result obtained by Professor Newcomb was the consequence of what Mr. Nevill deems the excessive weight given to the first eclipse of the series. The proper remark, therefore, to pass upon my second correction is that it is in accordance with Mr. Nevill. I do not pursue this branch of the inquiry, for observations considered by Professor Newcomb to have a probable error of 20 minutes of time or $600''$ are not to be combined with observations of solar eclipses where a residual of $100''$ would not be tolerated.

3. A correction of $+3''\cdot0$ to the secular term of the argument of latitude combined with $-1''\cdot6$ to the mean elongation produces $+4''\cdot6$ in the distance of the Sun from the node, on which depends the magnitude of the eclipse. Professor Newcomb has not discussed the magnitudes of the eclipses. In this connection I have made the following calculations :

Taking as a starting-point Professor Newcomb's table on p. 41 of his *Researches*, I have added or subtracted one-tenth of the difference in longitude to the Moon's latitude. In this